Comparative Study Concerning the Methods of Calculation of the Critical Axial Buckling Load for Stiffened Cylindrical Shells

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As demonstrated in numerous theoretical and experimental studies [1], the buckling behaviour of stiffened cylindrical shells (SCS) is strongly influenced by the presence of geometric imperfections caused by the manufacturing process and/or exploitation. Therefore, the design norms recommend the use of reduction coefficients with very low values, resulting in a significant reduction of the maximum load applied. In order to calculate the critical buckling load as accurately as possible it is necessary to know the real geometry of SCS. In case of SCS, the structural analysis based on the use of the finite element method (FEM), using models that reflect the real geometry of the shell determined from measurements, lead to a better evaluation of the critical buckling load. The structural analysis with FEM is accepted more and more by standards, EN 1993-1-6:2007 [2] specifying the types of numerical analysis accepted for cylindrical shells. The aim of this study is to compare the results concerning the critical buckling load for SCS under axial compression, obtained with both the analytical and FEM methods for real geometries obtained from measurements. For this purpose, scale models of SCS were used, for which were determined, by measuring, the values of the deviations from the median radius at several points on the shells surface. These deviations were then incorporated in the numerical analysis with FEM and it was determined, for each cylindrical shell, the value of the critical axial buckling load, by using geometrically nonlinear analysis. In order to validate the results of the numerical analysis, the analysed SCS were subjected to axial compression within an experimental program and the experimental data were compared with the results established on the basis of analytical and numerical calculation.

Keywords: buckling, stiffened cylindrical shell, geometric imperfections, axial compression, analytical solution, numerical analysis

Steel cylindrical shells are structures widely used in many fields, especially for the storage of liquids (tanks) and powdery or granular materials (silos) [3].

According to EN 1993-1-6:2007 [2], for tanks or silos structures, buckling under varying loads is the main criterion to be taken into account in their design.

The main types of loads that can lead to the buckling phenomenon for silos structures are axial compression due to the friction between the deposited material and the vertical walls or during seismic actions and external pressure due to the wind or the vacuum that takes place during the discharge of material inside these structures.

In general, these types of structures used in practice are provided with stiffeners that can be arranged axially and/or circumferentially. The presence of these elements may influence in a more or less significant manner the value of the critical buckling load, depending on the type of load to which the structure is subjected.

If the cylindrical shell is stiffened with a large number of stiffeners, the value of the critical axial buckling load or the critical external pressure can be determined using analytical models presented in [4, 5], namely [4, 6]. These solutions are described by the linear theory of structurally orthotropic shells.

However, if spacing between the stiffeners is not small enough, the correlation of the theoretical and experimental results obtained [7, 8] is found to be unsatisfactory. On the other hand, the presence of geometric imperfections is representing another aspect that is not considered in these analytical formulations, contributing to the increase of the differences between theoretical and the experimental results for the buckling of SCS.

For this reason, we have determined in this study the real geometry for three scale models of SCS and the measured geometric imperfections were incorporated into the finite element analyses. For the three models we have established the values of axial critical buckling loads, using geometric nonlinear numerical analysis (large displacements). These values were compared with those obtained in the experimental testing to the axial compression of the three models of SCS.

Specimens

In this study, three small-sized SCS were tested (fig. 1). It can be seen that the C1-8 and C2-8 models were stiffened with 8 longitudinal stiffeners and the specimen C3-16 was stiffened with 16 longitudinal stiffeners. Their dimensional features, determined on the basis of the similitude theory, and according to the size of a silo used for storing granular or powdery material, were:

- t = 0.5 mm-the wall thickness;
- R_i = 190 mm-the internal radius;
- H = 188 mm-the effective height.

All shells were stiffened with identical equidistant longitudinal thin-walled stiffeners of angle profile with dimensions 8 X 8 X 0.5 mm. Both cylindrical shells and stiffeners were made of stainless steel W4301 (X5CrNi18-10).

In order to determine the deviations from ideal geometry for the models studied, the experimental test facility presented in figure 2 was used. Meridional and circumferential lines were drawn on the shells external surfaces. The radial deviations were measured at the points that have resulted from the intersections of the meridional and circumferential lines, with a dial gauge of 0.01 mm precision. The values obtained were used to generate the numerical models used in the finite element analysis.

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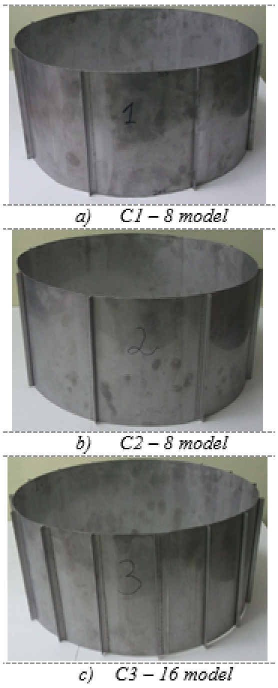


Fig. 1. Cylindrical shells models used in the study.

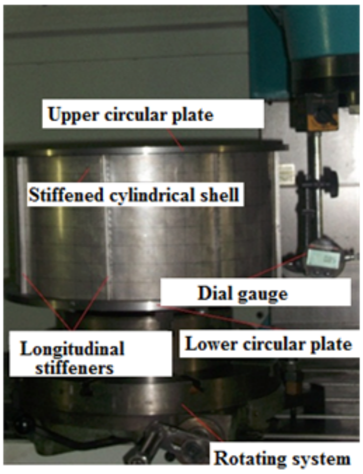


Fig. 2. Imperfections measuring system

Theoretical solution

The study concerning the buckling behaviour of small sized SCS under axial compression has involved both an analytical calculation and a numerical analysis using FEM. The calculations were made on both models with ideal geometry and real models for which there were measured the radial deviations in 8 X 48 points for the C1- 8 model and 16 X 96 points for the other two models used in the study.

Analytical solution

The analytical calculation for determining the critical axial buckling load for the analysed SCS models was made according to the methodology presented in EN 1993-4-1:2007 [9] where it is specified that, under axial compression, the wall should be designed for the same axial compression buckling criteria as the un-stiffened wall, unless the stiffeners are at closer spacing than $2\sqrt{Rt}$, where *R* is the radius of shell middle surface and *t* is the local thickness of the wall.

Considering the dimensional characteristics and the geometric configuration of the models used for the study, the analytical calculation was performed as for the un-stiffened shell, according to EN 1993-1-6:2007 [2]. The values of the critical axial buckling load for the models with perfect geometry $P_{cr}^{an,\,pg}$, namely for the models with deviations from the middle surface $P_{cr}^{an,\,impg}$ are shown in table 1.

Numerical analysis

In the numerical analysis, for each SCS, the finite element model was generated using SHELL93 elements type, both for the cylinder and the vertical stiffeners.

In order to model the real structure, with measured geometric imperfections, it was necessary to generate directly the finite element model by defining 8 X 48 nodes for the C1 - 8 model and 16 X 96 nodes for the other two models.

The geometric imperfections (the deviations from the median radius) of each considered shell were taken into account in the numerical linear and nonlinear analysis, by considering that the nodes of finite element model have the same coordinates (in a cylindrical coordinate system) as the measured points. In other words, the imperfect model is obtained by perturbing the radial coordinates of all nodes with the corresponding imperfection measured values.

Taking into account that, in situ, the possibility of measuring the deviations from ideal geometry is very difficult, we were looking for a method to evaluate the real median surface of the shell by using double Fourier series.

With the double Fourier series decomposition, we can represent the shell surface affected by geometrical imperfections that were measured in a finite number of points in the axial and circumferential direction.

The Fourier series used to represent the geometric imperfection take the following form [10]:

$$W(\theta,z)=\sum_{m=0}^{N_m}\sum_{n=0}^{N_n}\cos\left(\frac{2\cdot m\cdot\pi\cdot z}{L}\right)\left[A_{mn}\cdot\cos(n\theta)+B_{mn}\cdot\sin(n\theta)\right]+ \\ +\sin\left(\frac{2\cdot m\cdot\pi\cdot z}{L}\right)\left[C_{mn}\cdot\cos(n\theta)+D_{mn}\cdot\sin(n\theta)\right] \quad (1)$$

where N_m is the meridional harmonic number of the highest term adopted, N_n is the circumferential harmonic number of the highest term adopted; A_{mn}, B_{mn}, C_{mn}, D_{mn} - Fourier coefficients to be estimated; *z* - meridional coordinate on the shell; θ - circumferential coordinate on the shell (in radians); *L*- the total height of the shell.

In figure 3 are shown the values of deviations from median radius corresponding to the points located at z = 94 mm from the base of SCS in three situations: values measured in 16x96 points, values estimated with Fourier series by using 16 X 96 sampling points and the values determined with Fourier series by using only in 8 X 48 sampling points.

The graph in figure 3 indicates that the curves of variation of radial deviations have the same shape. When the number of sampling points is higher, the surface of SCS obtained from the decomposition in Fourier series is almost identical to the real shape. On the other hand, when using a smaller number of data points, the shape of the surface obtained is the same, but the values of deviations estimated are greater than in reality.

This is not an inconvenience in the analyses related to the buckling of SCS because the shape of real surface can b

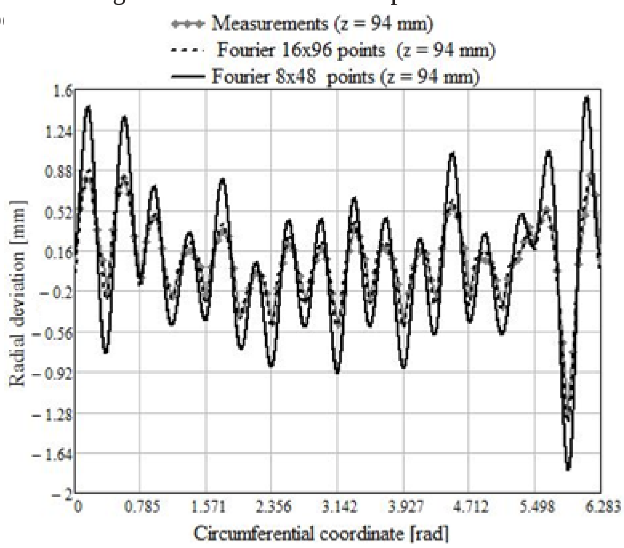


Fig. 3. Comparison of results based on Fourier series and the measured values of the radial deviations for the model C3-16 (z = 94 mm)

small number of data points, but the amplitudes of geometric imperfections estimated are bigger, which can lead to smaller values of the critical buckling loads than the real values. This aspect is covering the buckling point of view.

In the numerical analysis concerning the buckling of the real structures with geometric imperfections we have used both the linear-elastic analysis and the nonlinear analysis taking into account the large displacements (Geometrical Nonlinear Analysis with Imperfections).

The boundary conditions were imposed by the experimental assembly used to determine the critical axial buckling load shown in figure 6. The shells edges could not translate instead had free rotations.

Figures 4 and 5 show the critical buckling deformed shapes obtained when using the numerical linear analysis of structures with ideal geometry, respectively with nonlinear numerical analysis of structures with geometric imperfections determined from measurements.

Table 1 summarizes the results concerning the values of the critical axial buckling load obtained from the analytical calculation according to EN 1993-4-1:2007 [9], EN 1993-1-6:2007 [2] and numerically, for the three cylindrical shells analysed.

In this case, the critical axial buckling load for the imperfect shell P_{cr}^{impg} can be calculated as:

$$P_{cr}^{impg} = P_{cr}^{pg}\left(1-D_{imp}\right)^{\frac{1}{\alpha+1}} \qquad (2)$$

where P_{cr}^{pg} is the critical axial buckling load for the shell with perfect geometry, á depends on the behaviour of the material before buckling (for linear-elastic behaviour α = 1).

Thus, using the data presented in table 1, it can be calculated the value of the deterioration produced by the imperfections:

$$D_{imp} = 1-\left(\frac{P_{cr}^{impg}}{P_{cr}^{pg}}\right)^{\alpha+1} \qquad (3)$$

The results for the deterioration are shown in table 2.

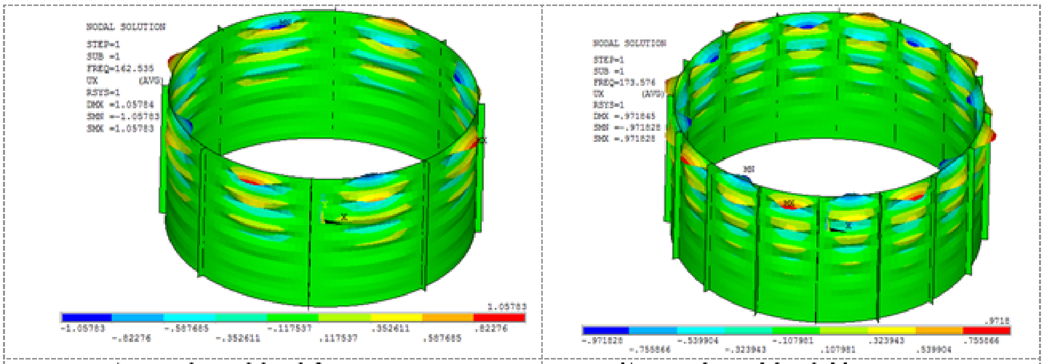
The results from table 2 show that the deterioration value is smaller for the shell with 16 stringers (C3-16) compared with those with 8 stringers (C1-8 and C2- 8).

It can be seen that the analytical solutions and the numerical results established with linear numerical analysis for the structures with perfect geometry are significantly close. If the deviations from the ideal geometry are taken into account, the use of linear numerical analysis leads to obtaining values of axial critical buckling load $P_{cr}^{num,\,l,\,impg}$ significantly higher than the

Table 1
THE INFLUENCE OF GEOMETRIC IMPERFECTIONS ON THE CRITICAL AXIAL BUCKLING LOAD

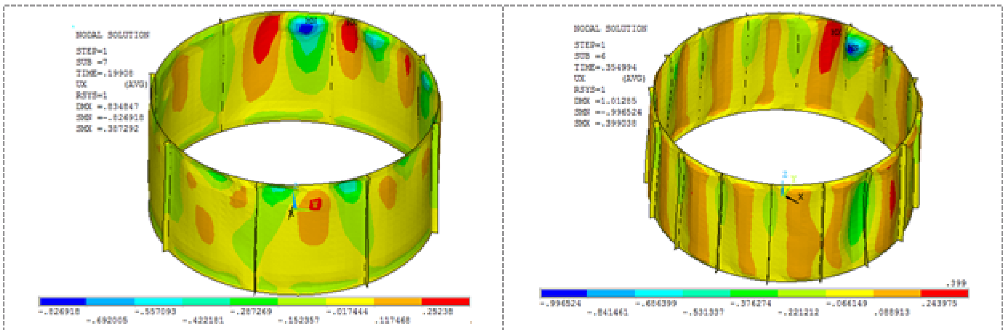
The model Analysed	**The value of critical axial buckling load, P_{cr} [N/mm]**				
	Numerical calculation			**Analytical calculation [2, 9]**	
	Linear analysis		Nonlinear analysis		
	The model with perfect geometry $P_{cr}^{num,\,l,\,pg}$	The model with geometric imperfections $P_{cr}^{num,\,l,\,impg}$	The model with geometric imperfections $P_{cr}^{num,\,nl,\,impg}$	The model with perfect geometry $P_{cr}^{an,\,pg}$	The model with geometric imperfections $P_{cr}^{an,\,impg}$
C1-8	162.53	97.76	45.19	159	27.65
C2-8		77.87	39.39		
C3-16	173.57	170.13	67.09		

According to the works [11 – 16], the shell imperfections can be assimilated with a deterioration D_{imp}.



a) *the model with 8 stringers* b) *the model with 16 stringers*

Fig. 4. The critical buckling deformed shapes obtained with numerical linear analysis for the ideal structures.



a) *C2 – 8 model* b) *C3 – 16 model*

Fig. 5. The radial displacements distribution obtained with nonlinear numerical analysis for the real structures (when reaching the critical buckling load)

Table 2
THE DETERIORATION PRODUCED BY THE IMPERFECTIONS FOR THE STIFFENED CYLINDRICAL SHELLS

The Analysed Model	The deterioration produced by the imperfections D_{imp}	
	Numerical	Analytical
C1 – 8	0.72	0.82
C2 – 8	0.75	
C3 – 16	0.61	

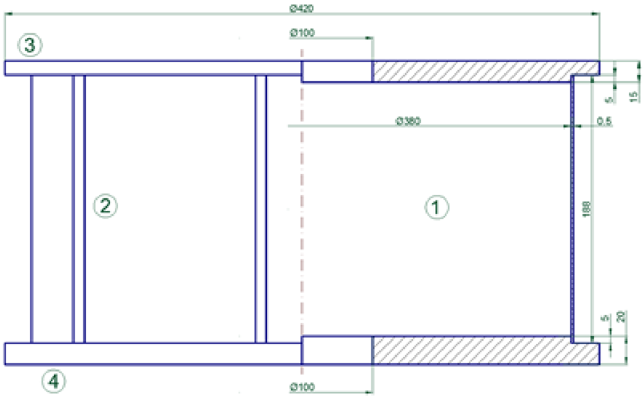


Fig. 6. Experimental assembly used for buckling tests of stiffened cylindrical shells. 1 - cylinder with 8 or 16 stringers; 2 -stringers; 3 -upper circular plate; 4 - lower circular plate

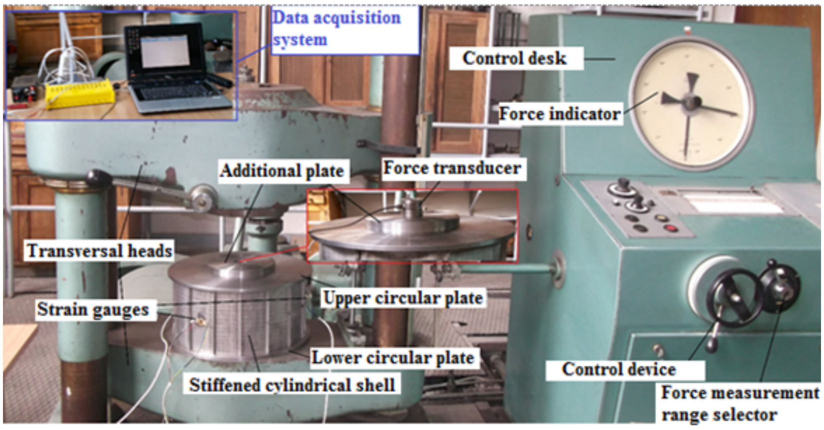


Fig. 7. Experimental device used for buckling tests of stiffened cylindrical shells under axial compression

values $P_{cr}^{an,\,impg}$ calculated using the standards. The nonlinear numerical analysis indicates that the presence of geometric imperfections decreases drastically the critical axial buckling load $P_{cr}^{num,\,nl,\,impg}$ (with 61 ... 76%) when compared to the ideal structure $P_{cr}^{num,\,l,\,pg}$. Unlike the analytical calculation that does not take into account the presence of stiffeners, the results of the nonlinear numerical analysis indicate a significant increase in the value of the axial critical buckling load $P_{cr}^{num,\,nl,\,impg}$ (48.46% ... 70.32%) for the model C3 - 16 that has 16 stringers, compared with the values corresponding to the models with 8 stringers (C1- 8 and C2 -8). Also, it can be observed that modelling the real (imperfect) geometry of the stiffened cylindrical shells analysed, the values of critical axial buckling loads obtained from nonlinear numerical analysis $P_{cr}^{num,\,nl,\,impg}$ are much lower (1.97 ... 2.53 times) than the values obtained with linear numerical analysis $P_{cr}^{num,\,l,\,impg}$, but are approximately 1.42 ... 2.42 times higher than the values $P_{cr}^{an,\,impg}$ obtained when applying the analytical calculation methodology described in the standards. These aspects illustrate on the one hand the conservative nature of analytical formulations and on the other hand the need to validate the numerical values obtained with nonlinear analysis based on FEM, validation that can be obtained by carrying out experimental tests.

Experimental part

Test procedure

In order to validate the use of the numerical method of calculation, the models of stiffened cylindrical shells above presented were tested in axial compression in the laboratory, by using the experimental assembly designed to determine the critical axial buckling load, presented in figure 6.

All cylindrical shells (C1- 8, C2- 8 and C3-16) were subjected to axial compression using the Universal Machine ZD 100. The experimental stand used is shown in figure 7.

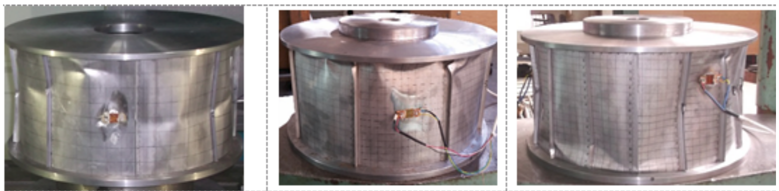
The axial compressive force was measured and recorded using a force transducer KMR 40 kN. In order to evaluate the stress in the wall of SCS, strain gauges 1-XY11–6/120 were used in two directions. The values of the measured parameters were recorded using the data acquisition system ESAM Traveller Static V2.0.

Test results

Figure 8 shows the buckling deformed shapes obtained experimentally for the analysed models. Table 3 gives the values of the axial critical buckling loads obtained from the experimental study, these values being compared with the values obtained from analytical calculations, respectively with FEM.

The results presented in table 3 indicate that for all three stiffened cylindrical shells, it can be observed that there is a very good agreement between the values of the critical buckling loads obtained from numerical nonlinear analysis for the models with geometric imperfections and the experimental ones. This justifies the use of numerical calculations in case of problems related to buckling of stiffened cylindrical shells subjected to axial compression.

It is evident that there are significant differences between the numerical and experimental results on the



a) C1 – 8 model   b) C2 – 8 model   c) C3 – 16 model

Fig. 8. The buckling deformed shapes obtained from experimental tests.

Table 3

THEORETICAL AND EXPERIMENTAL VALUES OF THE CRITICAL AXIAL BUCKLING LOADS

The analysed model	The value of axial critical Buckling load, [N/mm]			Differences [%]		
	Analytical $P_{cr}^{an,\,impg}$	Numerical $P_{cr}^{num,\,nl,\,impg}$	Exp. P_{cr}^{exp}	Numerical – Analytical	Experimental – Analytical	Numerical– Experimental
C1-8	27.65	45.19	43.29	63.43	56.56	4.39
C2-8		39.39	39.72	42.45	43.65	0.84
C3-16		67.09	59.72	142.64	115.98	11.23

one hand, and the results obtained with the analytical methodology presented in the standards [2, 9], on the other hand, this aspect illustrating once again the extreme conservative nature of the analytical calculation.

Conclusions

The determination of the values of the critical buckling load under axial compression for three small-sized stiffened cylindrical shells which geometry is affected by the presence of initial geometric imperfections was made both by using the analytical and numerical methods of calculation and the experimental analysis.

In the numerical analysis the real geometry of each cylindrical shell was modelled. This geometry was determined by measuring the deviations from the median radius.

It has been found that the use of double Fourier series to generate the median surface of SCS with imperfections based on a small number of data points leads to similar models, but the estimated amplitudes of imperfections are higher as the number of sample points is lower. So in a covering way, values of the axial critical buckling loads smaller than the real values can be obtained.

It has been revealed that the use of the numerical linear analysis leads to the overestimation of the axial critical buckling load, even when the deviations from ideal geometry are incorporated into finite element models, thus showing the need to use the nonlinear analysis in the numerical buckling analyses for SCS.

The influence of the vertical stiffeners has been highlighted by evaluating the value of the deterioration that would appear if the shell geometrical imperfections are assimilated to defects (table 2).

In order to validate the results of the numerical methods (FEM) on the stability of SCS axially compressed, experimental determinations were made.

It has been shown, both numerically and experimentally, that the use of a larger number of vertical stiffeners (16 stringers) increases significantly (even by 70%) the critical axial buckling load as compared to the case of cylindrical shells stiffened with 8 stringers, aspect that is not taken into account in the analytical calculation methodology presented in EN 1993-4-1:2007.

Very small differences have been obtained between the values of the axial critical buckling loads obtained from nonlinear numerical analysis for the real (imperfect) structure and experimental results (table 3). This justifies the use of numerical calculation for the problems concerning the buckling of stiffened cylindrical shells under axial compression, by using numerical models as close as size and shape of the real structures.

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